ROM-it! Mesh movement made cheap

by Alfred EJ Bogaers, Dr Schalk Kok and Dr Arnaud G Malan

A novel means was recently developed to reduce the computational costs of mesh movement. Mesh movement is a key component in fluid-structure interaction (FSI) simulations. This work was inspired by technology that is being developed by a team of researchers at the Council for Scientific and Industrial Research (CSIR).

Fluid-structure interaction (FSI) is the study of the interaction between fluid flow and deformable structures. Examples of FSI include the flow of air around a flexible aircraft wing, the opening of a parachute, or blood flow through the cardiovascular system. Studying the nature of the FSI of complex systems is a critical component in many engineering fields. It ranges from ensuring that an aircraft wing does not undergo drastic oscillations that will cause it to eventually break off from an aircraft, to understanding the internal mechanics of the human heart. Due to the ever-increasing power of central processing unit (CPU) technologies and the continuing maturity of numerical techniques, it is now possible to perform these simulations accurately using computer-based simulation models.

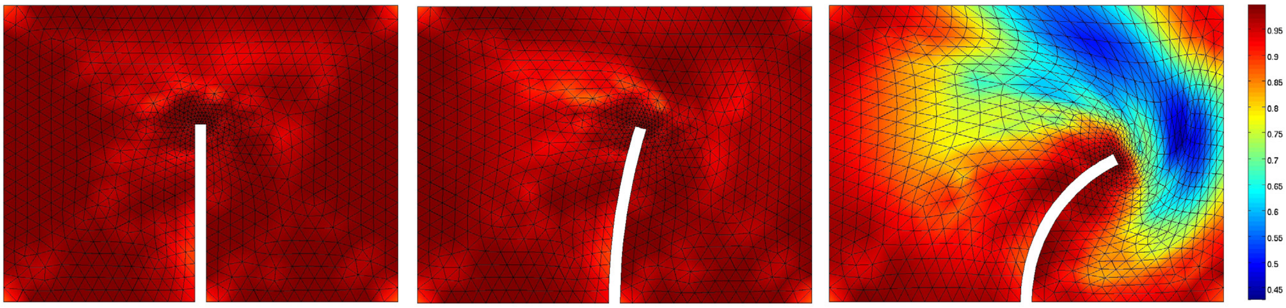
Unfortunately, due to the sheer size and complexity of these simulations, they are, in most cases, unfeasible unless performed on large super-computers or clusters. As such, researchers are continuously striving to decrease the associated cost. The research that was conducted culminated in a generic "black-box" technology that can be applied to essentially trivialise the costs associated with mesh movement. This is one of the critical components of an FSI simulation.

To provide some background – a computational mesh (grid) is the spatial discretisation required by numerical algorithms. The numerical algorithms solve a set of partial differential equations that mathematically describe the predominant physics at play. For example, in fluid flow, these equations may be used to describe and determine the fluid velocity, pressure and density as it is transported through the domain. In order to solve these differential equations, it becomes necessary for the spatial domain to be discretised, or broken into small cells or elements where the combination of these cells is known as the mesh. The numerical algorithm then solves the differential equations over each of the small cells using numerical techniques such as the finite volume or finite element method.

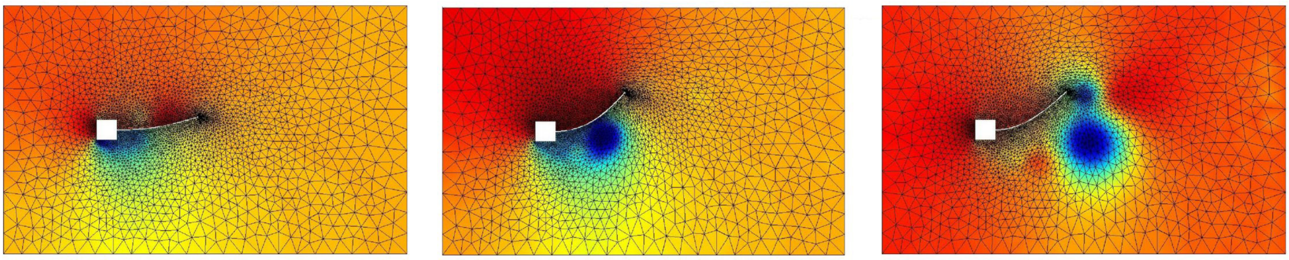
In FSI simulations, as the structure undergoes flow-induced deformations, it becomes necessary for the mesh to conform to the new displaced boundary. This is done either by regenerating the mesh at each instance of boundary deformation, or by moving the mesh itself. Regenerating a mesh presents several complications, including the fact that it is very expensive and that there is a possibility of destroying conservation, which in turn destroys the accuracy and stability of the simulation.

Mesh movement is a technique whereby the internal nodes of vertices of the mesh are relocated to accommodate the structural deformation. Several different mesh movement algorithms have been developed over the years. These are primarily aimed at reducing the frequency and necessity for mesh regeneration. The various techniques vary largely in terms of the associated cost, types and size of the boundary deformations they can support, and the quality of the resulting meshes after deformation. The stability and accuracy of the numerical scheme are strongly linked to the quality of the computational mesh.

Figure 1 and Figure 2 illustrate the concept of mesh movement. The figures are examples of common FSI benchmark problems used to study the accuracy of the numerical solution techniques. The problems entail thin elastic beams undergoing flow-induced oscillations. Despite the success of mesh movement methods to deform the computational mesh, they remain fairly expensive, and can account for a significant percentage of the total



→ 1. Example of mesh movement for a slender elastic beam undergoing first mode oscillations (the colour scale is an indication of mesh quality, where red implies a good element and blue a degenerate element).

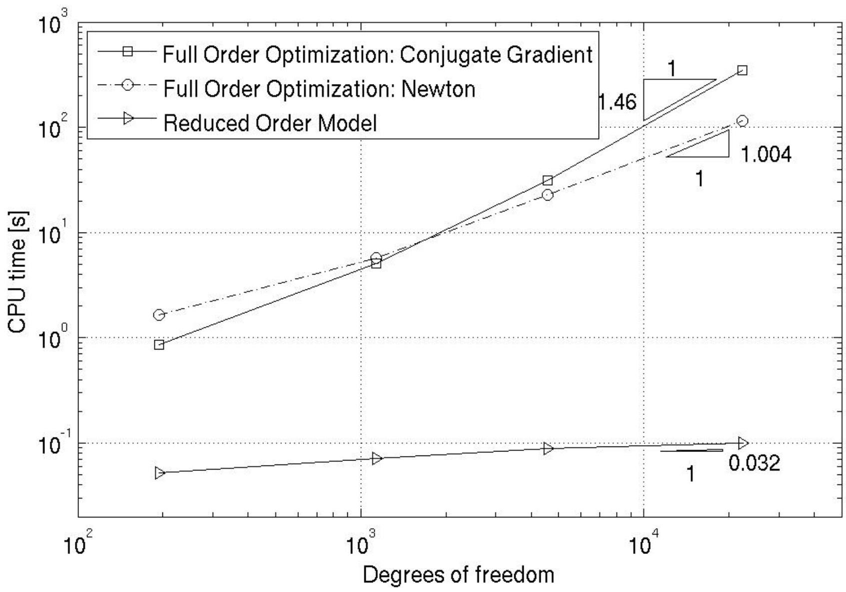


→ 2. Pressure plots for a common FSI benchmark simulation of flow over a fixed rectangle with a flexible elastic tail (the colour scale is an indication of pressure, where red and blue represent low and high pressures respectively).

CPU time required to perform a full FSI simulation. Depending on the complexity of the physics being solved, the total contribution of the mesh movement to the full costs can range from as low as 1% to as much as 50% of the full FSI simulation time.

The technique that was developed employs a reduced-order modelling (ROM) technique, known as proper orthogonal decomposition (POD). It is also commonly referred to as Kurhunnen-Loeve decomposition, principal-component analysis or singular-value decomposition. In essence, POD is a mathematical technique that allows a system to be decomposed into a subspace. In so doing, low-dimensional approximate descriptions of high-dimensional systems are generated.

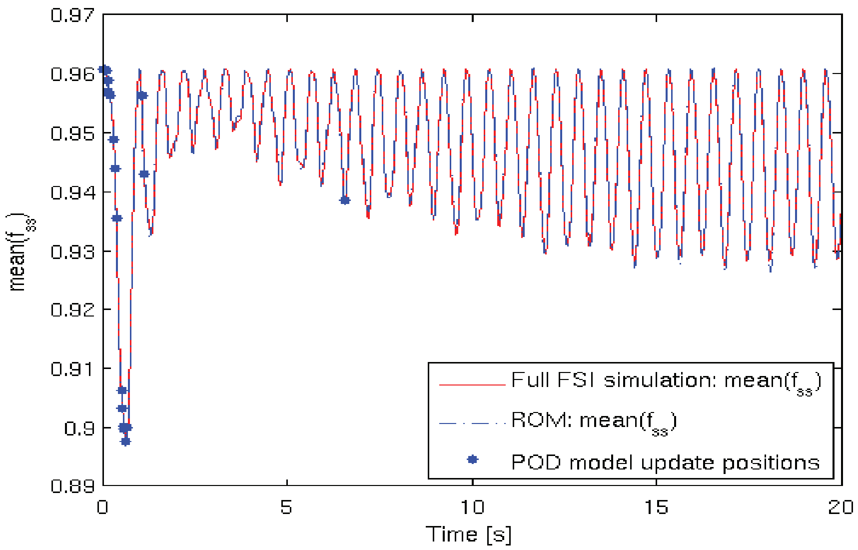
To get an idea of the potential savings offered by the implementation of POD to mesh movement, consider Figure 3, where a direct cost-scaling comparison is made between the POD model and



→ 3. CPU scaling, which compares the cost of full-order mesh movement, based on optimisation, to the cost of the POD-based ROM.

a mesh movement method based on optimisation. Not only is the POD model capable of reducing mesh movement costs, but it does so without any significant reductions in the quality of the original mesh movement scheme.

In order to develop an approximate model of a system using POD, the model has to be trained. The training process involves gathering a set of "snapshots" of the system, where each snapshot presents the full solution of



4. Plot of the mean mesh quality for the benchmark problem of the oscillating flexible tail depicted in Figure 2 for 4 500 time steps: the mesh quality is measured using a quality metric defined so that a perfect and degenerate mesh attains a value of 1 and 0 respectively (the mesh movement is performed using both the full-order mesh movement and the adaptively trained POD-based ROM).

the system for a given set of inputs. Applied to mesh movement, these snapshots present the mesh solution of the full-order mesh movement scheme for different boundary deformations. The POD model is then capable of reproducing information that closely resembles the information contained in the training snapshots.

Despite POD being able to provide exceptional approximations of systems at a mere fraction of the original cost, its limiting factor is the fact that it requires training. In the context of mesh movement, it is not particularly useful if a large set of expensive computations is necessary to develop a cheaper model, especially if the system parameters (such as the number of snapshots or the type of deformations the system may have to undergo) are not known prior to a simulation. To circumvent this problem, a novel adaptive training procedure was developed.

The training procedure allows the POD model to be trained through the course of a simulation only as it becomes necessary. Figure 4 illustrates the potential of the adaptive model. The plot shows the mesh quality produced by both a full-order mesh optimisation movement method and that of the adaptively trained POD model. The mesh quality is for the beam that is undergoing oscillatory motion across a total of 4 500 time steps in Figure 2.

In total, the adaptive POD model requires only 16 updates of snapshots, where each of these updates uses a single full-order mesh movement solution. This translates into a reduction in mesh movement costs for this particular problem from 4 500 full mesh movement computations to just 16. The negligible cost of the POD model implies time savings of three orders of magnitude, with essentially no loss in the produced mesh quality.

Overall, the work constitutes a distinct step towards making FSI technology computationally more affordable. The developed technology shows great potential, with a distinct possibility of being adopted into future FSI simulation codes.

References

1. Bogaers, A.E.J. 2010. Reduced Order Modelling Techniques for Mesh Movement Strategies as Applied to Fluid Structure Interactions. Master's thesis. South Africa: University of Pretoria.
2. Oxtoby, O., Malan, A.G. 2009. A Unified Finite-Volume Approach to Fluid-Structure Interaction. 15th International Conference on Finite Element in Flow Problems (FEF09). Japan: Tokyo.

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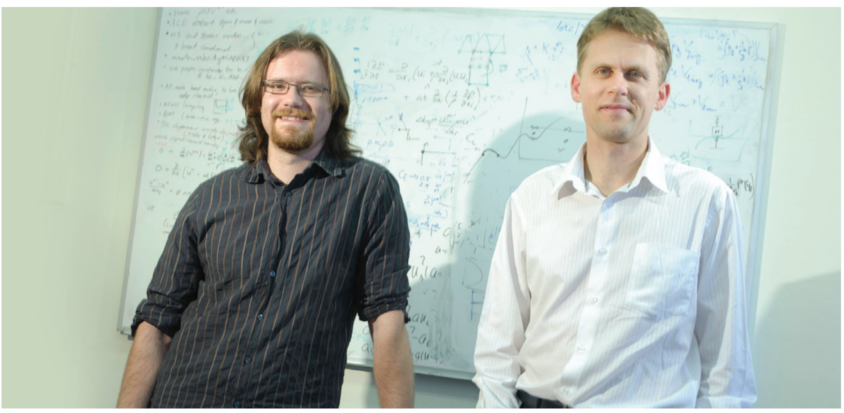
***Dr Schalk Kok** and **Dr Arnaud Malan** are principal researchers at the CSIR. (Dr Kok was not available for a photograph.)*



Alfred Bogaers (left) and Dr Arnaud Malan of the CSIR have developed a novel means to reduce the computational costs of mesh movement.