

Predicting the buckling length of compression chords in prefabricated roof structures

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In South Africa, timber-trussed roof structures supporting concrete tiles have for many years often been braced solely by diagonal braces. Failures have shown that the diagonal brace was inadequate for larger span roofs, and the use of diagonal bracing has subsequently been limited to spans of less than or equal to 10 metres.

When designing the compression chords of a timber truss in a braced roof, South African National Standards (SANS) 10163:1 (2003) recommends a minimum effective length for out-of-plane buckling of no less than $15 \times b$, which is 540 mm for a 36 mm-wide member. This effective or out-of-plane buckling length of the top chord was later assumed to be equal to the spacing of the trusses.

With the availability of PC-based packages that can perform three-dimensional buckling analyses, it is useful to investigate the validity of using the effective length equal to the truss spacing, and then also the 10 m limit on span for roofs braced by diagonal braces.

Since the introduction of computer-based analysis programs for timber roofs, the pre-processing and post-processing parts of the software have changed and improved to the extent that the designer is no longer aware of the design process, even though forces, displacements, sizes and assumed effective lengths may be printed for checking by a competent person.

Loads are calculated from the layout and these are applied to a two-dimensional analysis of the truss, even though a timber roof structure is a three-dimensional problem constructed of a brittle material with limited ductility in the connections.

Limited ductility can be a problem in cases where construction errors have been made and force-fitting is applied. Assumptions are made about the member sizes and sometimes also about the connector plate stiffness for an initial analysis. More often than not, a centreline analysis using

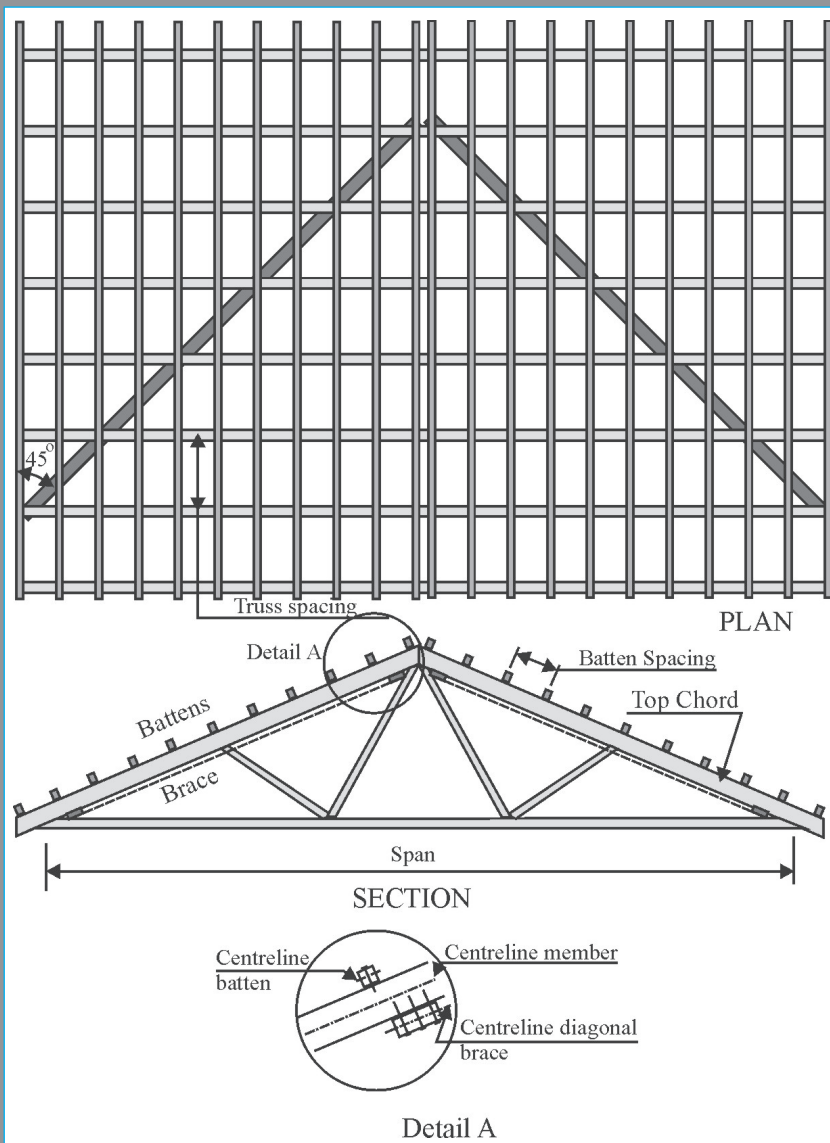
beam elements is used and the forces obtained in this way are used to size the members in accordance with SANS 10163: Part 1 (2003) or Part 2 (2001).

At this stage of the design, assumptions are made about the type of bracing to be used, as well as the effective or buckling length of the compression chord that would result from using that specific type of bracing. Many believe that the effective or buckling length of the top chord is equal to the spacing of the battens. This assumption would perhaps be correct if the tiles could be relied on to supply diaphragm action, and if the battens were rigidly connected to the compression member. In such a case, any further bracing would only be required for erection purposes. Diaphragm bracing by the tiles will initially be active, but with time, the friction between the tiles seems to break and movement occurs. This eventual movement of the tiles has led to the failure of roofs.

The assumption of the buckling length equal to $15 \times b$, with $b = 36$ mm, may not be a problem when the spacing of the trusses is equal to

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640 mm, as is common in Australia, but could become a problem where the spacing of the trusses is as much as 1 050 mm, as is often found in South Africa. The minimum slenderness of $L_e/b = 15$ was later changed by the South African Institute for Timber Construction to an in-house rule, which suggests an effective buckling length of the spacing of the trusses (750 mm to 1 050 mm).



→ Figure 1: Positioning of diagonal braces.

It is believed that a blanket rule such as effective length = $15 \times b$ or even buckling length equal to the spacing of the trusses may not be conservative, as the buckling length depends on the boundary conditions, the stiffness of the bracing and the method of transferring loads once buckling is initiated.

For small-span timber trusses (up to 10 m), a diagonal brace is the norm in South Africa. As the limit on the span for the use of a diagonal brace is less than 10 m, only the 10 m

and 7.5 m spans were investigated. Timber sizes for 10 m span roofs would typically be 36 mm x 111 mm top and bottom chords, with 36 mm x 73 mm web members. When bracing a 10 m-span trussed roof, a 36 mm x 111 mm timber member is fixed to the underside of the compression chords and runs at about 45° when seen in plan. Three 100 mm long nails are used to fix the brace to the underside of the top chord (Figure 1). Maximum spacing rules are used to ensure that trusses are not too far from the brace. Battens, the smallest nominally being

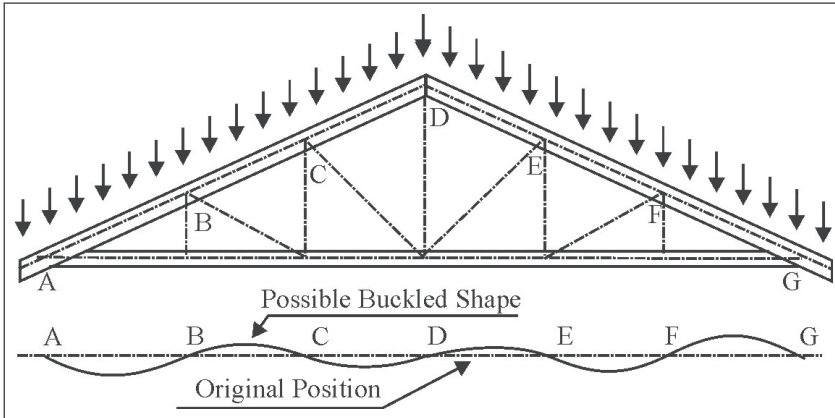
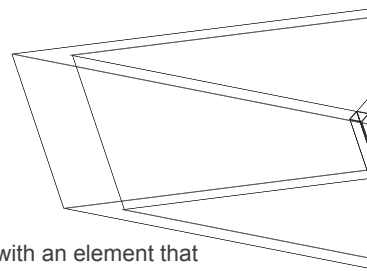
36 mm x 36 mm, are placed on top of the compression chord. These are then fixed to the compression chord with one 75 mm long nail.

A study was conducted to investigate diagonal bracing and all forms of bracing that are currently used by the South African timber roof truss industry. Although this investigation was a theoretical exercise and could not be validated by test results, it can be concluded that a three-dimensional buckling analysis is an acceptable way of determining the buckling length of a compression chord in a timber roof structure. Buckling and finite element analyses are widely used for many structural systems and materials, as the analyses are based on theories that have historically been proven to work for structures.

Effective length factors in simple lattice structures

The effective length factor is used to adjust the actual unrestrained length of a compression member to account for prevailing boundary conditions. Many software packages use a default out-of-plane effective length factor of 0.85, implying some form of rotational joint restraint by adjacent members. This is only possible where adjacent members have high out-of-plane bending or torsional stiffness and are themselves not compression members that could buckle. Some design codes specify effective length factors for compression members in lattice trusses.

Boundary conditions that influence the degree of restraint exercised on a compression member are not merely a function of connection details and continuity, but are influenced by the capacity of adjacent members at the node. Consider the example of a simple lattice truss with a constant section (Figure 2), where lateral supports are provided at seven nodes. The compression chord is divided into



→ Figure 2. A possible buckled shape of the top chord.

equal portions. The basic principle that the buckling load is unique shows that an effective length factor of less than one for a particular member is consistent with an effective length factor of greater than one in the adjacent members, albeit with a smaller force.

Stiffness of connectors

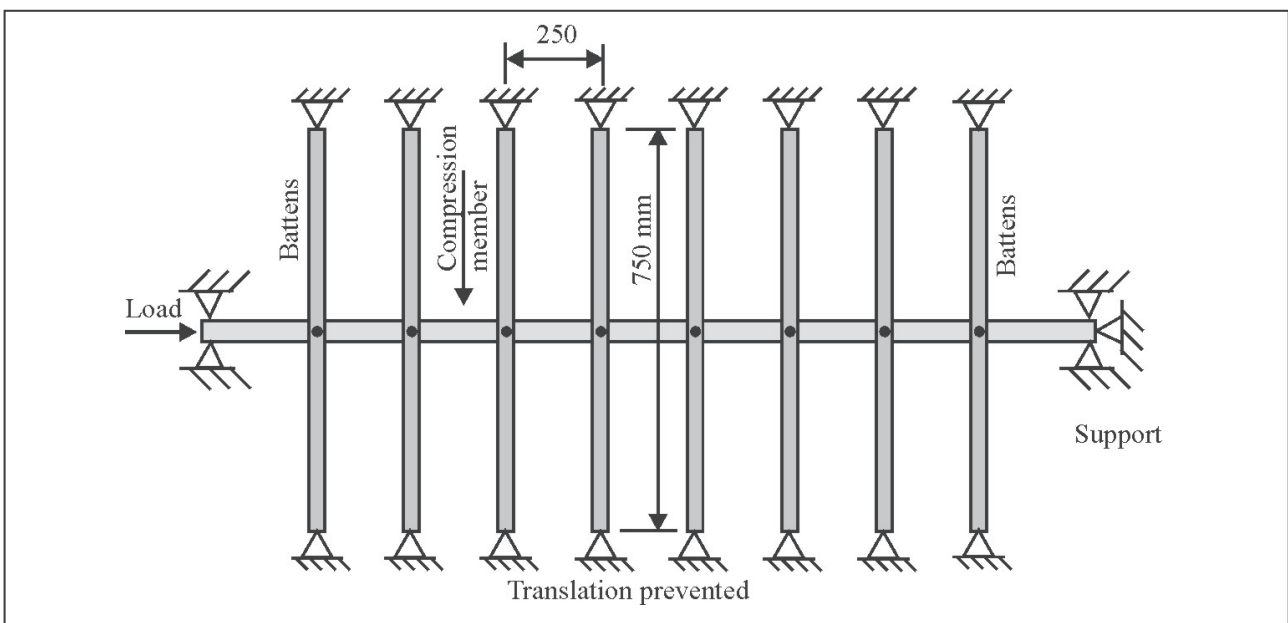
When a stiffness matrix method with beam elements is used to analyse a

structure, there are a number of ways of modelling the connection between, for instance, the batten and the top chord. One of the methods is the use of a spring as a connector. This, however, does not adequately address the possible rotation of the top chord, as the chord, the spring and the batten are in the same plane. Rotation of the chord will then 'soften' the stiffness of the connection. If a beam element analysis is applied to the three-dimensional model, it would be better

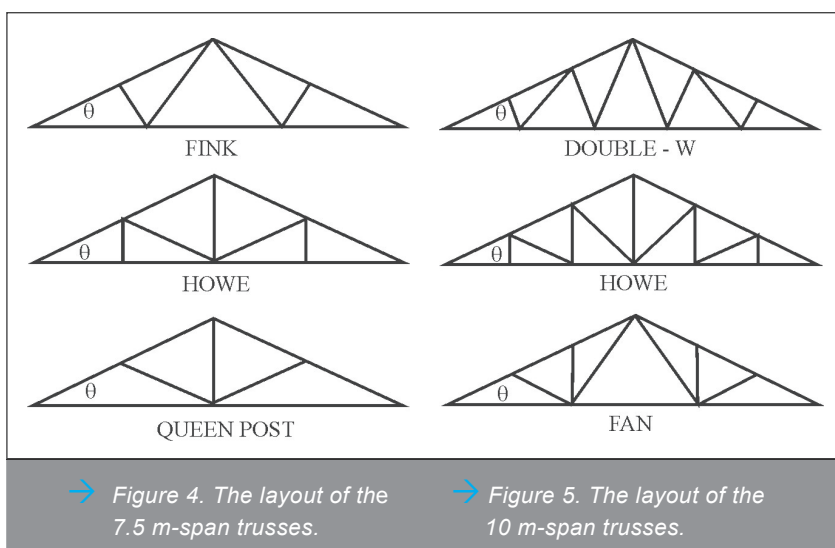
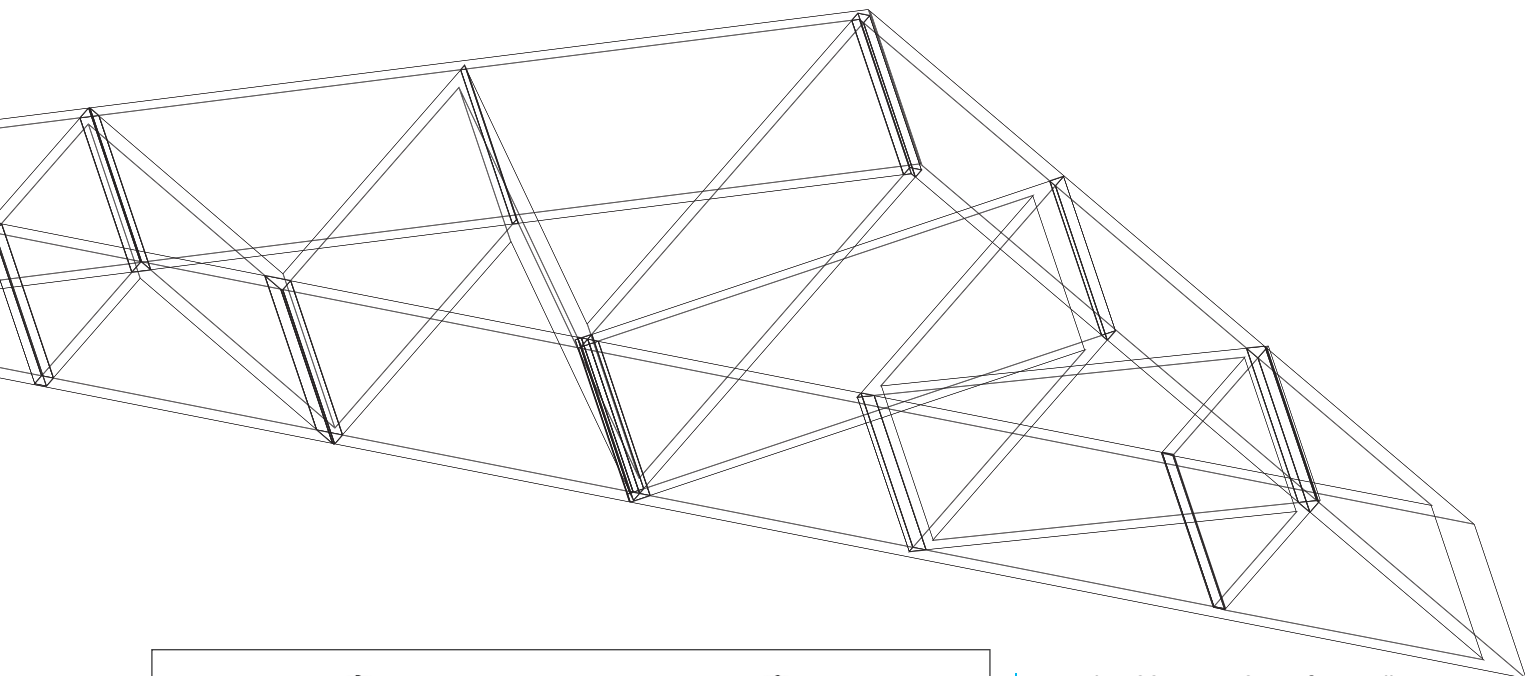
to model the nail with an element that has the same bending stiffness as the transverse stiffness of the nail in double curvature, than to use a spring. The spring will not have the necessary eccentricity to allow the torsional displacement of the chord.

To illustrate how the connection 'softens' with relative rotation between two connected members, two analyses were undertaken (Figure 3).

In the first analysis, the eccentricity between the centreline of the battens and the compression element was taken into account, with the nails being modelled by bending elements. No account was taken of the possible lower shear modulus of the compression member. A buckling load factor of 174 and an out-of-plane buckling length of 622 mm resulted. In the second analysis, the member was analysed using shell elements with springs connecting it to the battens. The spring stiffness was reduced until the same buckling factor (174) was obtained as in the first analysis. The spring stiffness that was



→ Figure 3. Theoretical set-up to investigate softening of the stiffness of the connections between battens and the braced member.



required to achieve this was 267 kN/m and no longer 800 kN/m. This shows that, when using shell or plate elements that are connected to the bracing battens by way of springs, great care should be taken, as it may result in misleading answers.

Shear modulus and analyses

The shear modulus of South African pine is accepted to be about equal to MOE/13. In order to demonstrate the principles discussed above, a commonly available PC-based analysis package (Prokon) was used to calculate the effective length factors of the top chord of gable-to-gable timber-trussed roofs with spans of 7.5 m and 10 m and pitches of 17.5°, 25° and 35°. The batten spacing was taken as 262 mm and 305 mm respectively in order to simplify the input of the truss and batten geometry. Only the tile weight and the self-weight of the timber were used to determine

the buckling length of the top chords, as the buckling is a long-term problem, rather than a problem that occurs only when imposed load is applied, as imposed load will increase the friction between the tiles, leading to bracing by diaphragm action. Tile mass was taken as being 55 kg/m², although the actual mass is not that important, as the buckling analysis is only used to calculate buckling lengths.

The different configurations were used to ascertain whether the configuration would play a significant part in the buckling length of the compression chord.

In all cases, the top and bottom chords were assumed to have dimensions of 36 mm x 111 mm, with web members being 36 mm x 73 mm with a 36 mm x 111 mm diagonal brace, although in practice the top and bottom chords may be 36 mm x 73 mm and the diagonal bracing

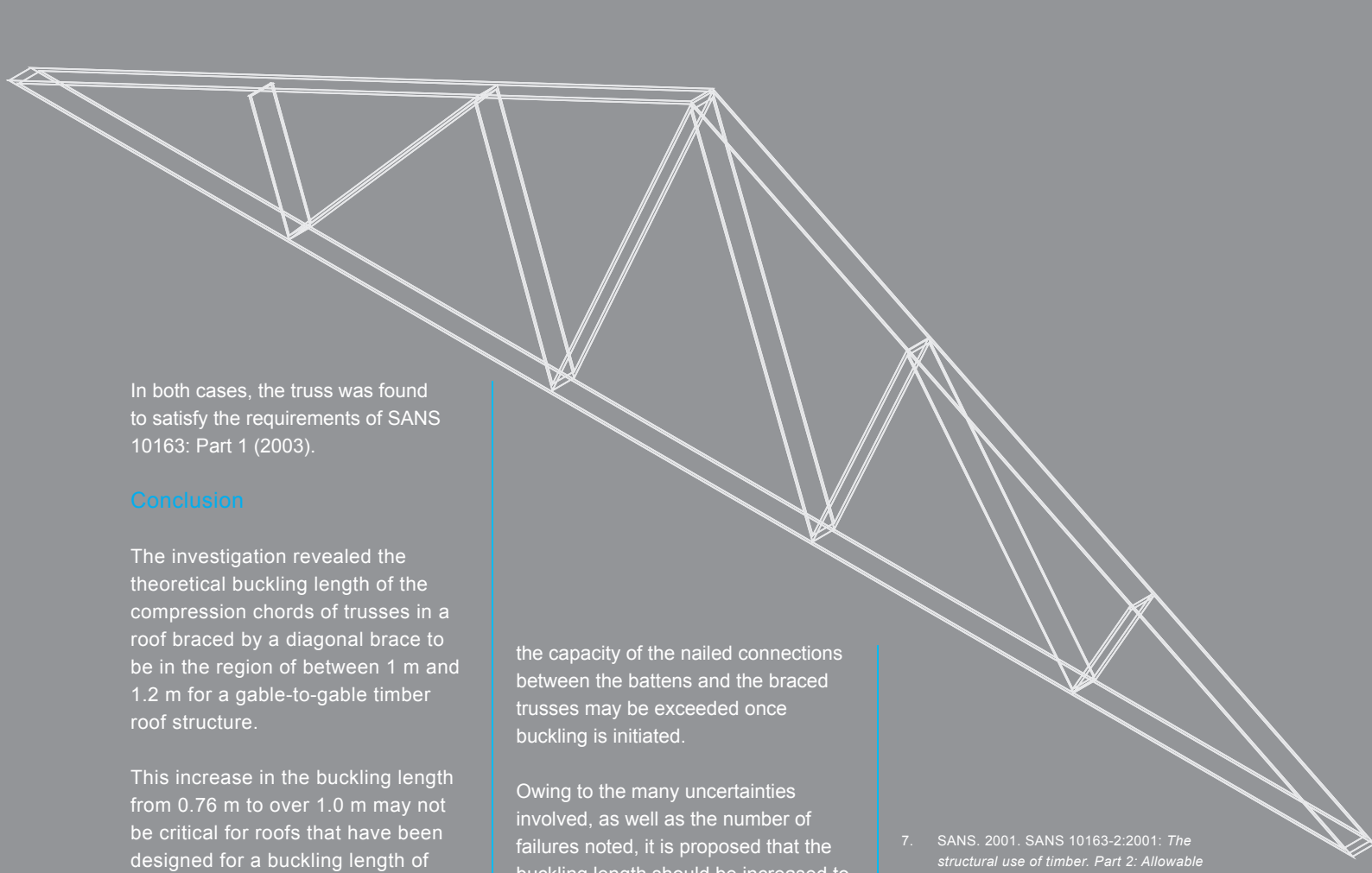
member 36 mm x 73 mm for small-span roofs.

A full-span complete roof was analysed to ascertain the buckled shape of the roof so that a half-structure, with the correct boundary conditions, could be analysed. From the buckled shape, one can deduce that the apex moves as the brace is flexible and it then becomes apparent that one cannot assume an inflection point at the apex. This then makes it possible to define the boundary conditions for a structure where only the half-structure is investigated. If the half-structure with the correct boundary conditions is used, it simplifies the input and speeds up the analyses of the various truss layouts and spans.

When comparing the results of the analysis for the different configurations, it is clear that the actual buckling length exceeds the purlin spacing by a factor of between 3.8 and 4.4. The error caused by centreline modelling was undertaken, as it was assumed that the difference in the buckling factors for the different layouts would be insignificant.

Ultimate strength of trusses

To see whether the theoretical increased buckling length would negatively influence the design of timber trusses, one of the 7.5 m trusses and one of the 10 m trusses were used to illustrate the code requirements between using the effective buckling length based on the truss spacing and the theoretical buckling length of 1.2 m.



In both cases, the truss was found to satisfy the requirements of SANS 10163: Part 1 (2003).

Conclusion

The investigation revealed the theoretical buckling length of the compression chords of trusses in a roof braced by a diagonal brace to be in the region of between 1 m and 1.2 m for a gable-to-gable timber roof structure.

This increase in the buckling length from 0.76 m to over 1.0 m may not be critical for roofs that have been designed for a buckling length of 0.76 m, or the spacing of the trusses, as the imposed load is very seldom applied to the full roof.

Furthermore, the imposed load would increase the friction between the tiles, perhaps leading to diaphragm bracing. The 30% shortfall in capacity should not impact significantly on the probability of failure of the compression chords, provided that the integrity of the connections between the trusses and the battens is maintained.

Ignoring the lack of torsional stiffness of the top chord has a small effect on the buckling length obtained from the analysis. This may not be true for sections that have a greater depth (149 mm and 225 mm). However, a centreline analysis that neglects to consider the distance between the centrelines of the brace, the chords and the battens is shown to underestimate the theoretical buckling length by a dangerous margin, possibly leading to unsafe member sizes.

The buckling analyses and calculations would appear to justify limiting the span of trusses that are braced by a diagonal brace to less than 10 m, as

the capacity of the nailed connections between the battens and the braced trusses may be exceeded once buckling is initiated.

Owing to the many uncertainties involved, as well as the number of failures noted, it is proposed that the buckling length should be increased to 1.2 m or $30 \times b$ for timber-trussed roofs that are braced solely by diagonal bracing. Perhaps there should be two interaction equations for checking the lateral buckling strength of the roof trusses. The first check should be to ascertain whether the truss strength is adequate for permanent load with the increased buckling length, and the second for total load, with the buckling length, however, reduced to $15 \times b$. ❄

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